



Electron and muon electric dipoles in supersymmetric scenarios

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Abstract

We study if a sizeable muon electric dipole can arise in supersymmetric frameworks able to account for the tight experimental bounds on sfermion masses, like an appropriate flavor symmetry, or like a flavor-blind mechanism of SUSY breaking (in presence of radiative corrections characteristic of GUT models, or due to Yukawa couplings of neutrinos in see-saw models). In some cases it is possible to evade the naïve scaling $d_\mu/d_e = m_\mu/m_e$ and obtain a d_μ as large as $10^{-22 \div 23} e \text{ cm}$. In most cases d_μ is around $10^{-24 \div 25} e \text{ cm}$ and $(d_\mu/d_e)/(m_\mu/m_e)$ is only slightly different from one: this ratio contains interesting informations on the source of the dipoles and on the texture of the lepton Yukawa matrix. We also update GUT predictions for $\mu \rightarrow e\gamma$ and related processes.

1 Introduction

The electric dipole moments (EDMs) of elementary particles represent a powerful probe of physics beyond the Standard Model (SM). In the SM, once the strong CP problem has been taken care of, the EDMs are far beyond reach of foreseeable experiments, whereas supersymmetric extensions of the SM may provide predictions in the range of interest. Whereas the experimental [1, 2, 3, 4] and theoretical [5, 6, 8, 9, 10] interest have mainly focussed on the EDMs of the electron and light quarks (through neutron and atomic EDMs), the recent prospects of improving the sensitivity to the muon EDM d_μ by 5 orders of magnitude down to $d_\mu \sim 10^{-24} e \text{ cm}$ [11] make d_μ an additional observable of interest.

We focus our attention on supersymmetric extensions of the SM, that remain the relatively more promising solution of the Higgs mass hierarchy ‘problem’ [12], although no sparticles have been found at LEP. The MSSM (i.e. the softly broken supersymmetric extension of the SM with minimal field content and unbroken R -parity) potentially contains additional sources of CP-violation associated to the supersymmetry breaking part of the Lagrangian. With no assumptions on it, it is of course possible to find a region in the huge parameter space that gives a detectable d_μ . A complex muon A -term, A_μ , is the simplest possibility. Ref. [13] discusses additional possibilities.

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On the other hand, the supersymmetry breaking parameters are strongly constrained by the necessity to avoid too large CP-violating and flavor-violating effects in the K and B systems, in μ decays, and in the electron and neutron EDMs. We therefore discuss in this paper whether a detectable d_μ can arise in frameworks able to account for the constraints on the structure of supersymmetry breaking, like a flavor blind mechanism of SUSY breaking or some appropriate flavor symmetry. In the most constrained situation with complex universal soft terms one has $A_\mu \approx A_e$ and $m_{\tilde{e}} \approx m_{\tilde{\mu}}$ giving $d_\mu \approx +d_e m_\mu/m_e$, necessarily a factor 3 below $10^{-24}e\text{cm}$ given the present limit on d_e [2]. More generically, it is well known [5] that a small phase in flavor-conserving soft terms (like gaugino masses, μ and $B\mu$ terms) can give a d_e just below its experimental bound and a $d_\mu \approx d_e m_\mu/m_e$. We will instead consider supersymmetric scenarios where CP-violation must be accompanied by flavor violation (similarly to what happens in the SM), so that large CP-violating phases give acceptable dipoles that evade the naïve scaling.

One way to account for the constraints on the flavor structure of supersymmetry breaking is that the supersymmetry breaking mechanism itself generates flavor-universal real soft terms at some scale M_0 . Even in this case, significant effects can arise if the theory above some scale M_{CP} lower than M_0 contains additional sources of CP-violation. Such effects leave their imprint in the soft terms through radiative corrections arising between the scales M_{CP} and M_0 [6]. Even if $M_{\text{CP}} \gg M_Z$ and the source of CP-violation decouples below M_{CP} , CP-violation survives at lower energies in the soft terms. In section 2 we study the effects due to SU(5) or SO(10) unification on a spectrum universal at the Planck scale. In section 3 we consider the effects due to the neutrino Yukawa couplings in the context of the see-saw mechanism. In section 4 we discuss unified see-saw models. The muon EDM in a left-right symmetric see-saw model has been studied in [7].

In section 5 we will consider the alternative possibility that the same physics accounting for the structure of fermion masses and mixings also determines the structure of sfermion masses. Suitable flavor symmetries can force a non flavor-universal, but phenomenologically acceptable, pattern of sfermion masses. Even assuming real A -terms (as suggested by the bounds on EDMs [1, 2, 3]), the complex lepton Yukawa matrix gives rise to a non trivial pattern of CP-violating effects.

The results are summarized in section 6. A muon EDM as large as $10^{-22 \div 23}e\text{cm}$ can be naturally obtained in a few cases but none of the scenarios we consider guarantees that. On the other hand, an electron EDM and a $\mu \rightarrow e\gamma$ rate within the sensitivity of planned experiments [8, 9, 10] are a prediction of some of the scenarios mentioned above. If supersymmetry and d_e were discovered, a measurement of d_μ sufficiently accurate to distinguish $d_\mu/m_\mu \approx +d_e/m_e$ from e.g. $d_\mu/m_\mu \approx -d_e/m_e$ would provide interesting information on the source of the dipoles, and, eventually, on the 11 element of the lepton mass matrix, allowing to test interesting (but so far only theoretical) speculations about flavor. Such a measurement needs a sensitivity to the muon EDM at least as low as $10^{-25}e\text{cm}$. In longer terms, this level of sensitivity could be reached using the intense muon beam produced at a future neutrino factory complex [11].

2 Effects from unified models

In unified models, flavor and CP violations cannot be confined to quarks but must be present also in leptons, e.g. $b \rightarrow s\gamma$ must be accompanied by $\mu \rightarrow e\gamma$. In non supersymmetric GUT models these leptonic effects are negligible because suppressed by powers of $1/M_{\text{GUT}}$. In supersymmetric GUT models with soft terms already present above the unification scale, quantum corrections due to the unified top quark Yukawa coupling imprint lepton flavor and CP violations in the slepton mass terms [6], inducing significant computable effects in low energy processes [8, 9].

Minimal SU(5) unification induces an electric dipole of the up quark, $d_u \propto \tan^4 \beta$, of experimental interest if $\tan \beta$ is large, but does not give detectable lepton electric dipoles [14]. On the other hand, minimal SO(10) unification gives $\mu \rightarrow e\gamma$ rates $(m_\tau/m_\mu)^2$ -times larger than in SU(5), and significant

present bound				future?	present bound				future?
d_N	$<$	$6.3 \cdot 10^{-26} e \text{ cm}$	[1]	$10^{-27} e \text{ cm}$	$\text{BR}(\tau \rightarrow \mu \gamma)$	$<$	$1.1 \cdot 10^{-6}$	[15]	10^{-9}
d_e	$<$	$1.5 \cdot 10^{-27} e \text{ cm}$	[2]	$10^{-29} e \text{ cm}$	$\text{BR}(\mu \rightarrow e \gamma)$	$<$	$1.2 \cdot 10^{-11}$	[16]	10^{-14}
$d_{199\text{Hg}}$	$<$	$1.8 \cdot 10^{-28} e \text{ cm}$	[3]		$\text{BR}(\mu \rightarrow e \bar{e} e)$	$<$	$1.0 \cdot 10^{-12}$	[17]	10^{-16}
d_μ	$<$	$1.0 \cdot 10^{-18} e \text{ cm}$	[4]	$10^{-25} e \text{ cm}$	$\text{CR}(\mu \rightarrow e \text{ in Ti})$	$<$	$6.1 \cdot 10^{-13}$	[18]	10^{-18}

Table 1: *Compilation of 90% CL bounds on CP-violating and lepton-flavor violating processes.*

CP-violating effects [9, 10]. SO(10) effects in $\mu \rightarrow e \gamma$, d_e and d_μ are correlated by

$$d_e = \text{Im } d_{ee}, \quad d_\mu = \text{Im } d_{\mu\mu}, \quad \text{BR}(\mu \rightarrow e \gamma) = \frac{m_\mu^3}{16\pi\Gamma_\mu} (|d_{e\mu}|^2 + |d_{\mu e}|^2), \quad (1)$$

where

$$d_{\ell\ell'} = F \times V_{\ell_R \tilde{\tau}_R} V_{\ell'_L \tilde{\tau}_L} V_{\tau_L \tilde{\tau}_L}^* V_{\tau_R \tilde{\tau}_R}^* \quad (2)$$

so that $d_{e\mu} d_{\mu e} = d_{ee} d_{\mu\mu}$. The common loop factor F is explicitly given in appendix B. In the same way as the CKM matrix measures the flavor misalignment within the $\text{SU}(2)_L$ multiplet of left-handed up and down quarks, the $V_{\ell_L \tilde{\ell}_L}$ ($V_{\ell_R \tilde{\ell}_R}$) mixing matrices measure the flavor misalignment within the supermultiplet of left-handed (right-handed) leptons and left-handed (right-handed) sleptons. In the usual supersymmetric flavor basis in which the charged lepton mass matrix is diagonal, these matrices diagonalize the left-handed (right-handed) slepton mass matrices.

Minimal SO(10) (defined as in [10]) predicts equal left and right-handed lepton/slepton mixing, so that $\text{BR}(\mu \rightarrow e \gamma) = m_\mu^3 |d_{ee} d_{\mu\mu}| / (8\pi\Gamma_\mu)$. Furthermore, minimal SU(5) and SO(10) models predict the unification of the lepton and down-quark Yukawa matrices at M_{GUT} , which would imply

$$|V_{e_L \tilde{\tau}_L}| = |V_{e_R \tilde{\tau}_R}| = |V_{td}|, \quad |V_{\mu_L \tilde{\tau}_L}| = |V_{\mu_R \tilde{\tau}_R}| = |V_{ts}| \quad \text{renormalized at } M_{\text{GUT}}. \quad (3)$$

However, it is well known that the corresponding minimal GUT relations between m_e, m_μ, m_d, m_s are wrong by factors of 3. This problem can be easily (and maybe even nicely [19]) solved in non minimal GUTs¹ but in practice implies that relations (3) are also wrong by unknown $\mathcal{O}(3)$ Clebsh factors. Setting these factors to one we would find

$$\frac{d_e}{d_\mu} \approx \frac{V_{td}^2}{V_{ts}^2} \quad \text{and} \quad d_\mu \lesssim 10^{-25} e \text{ cm} \sqrt{\frac{\text{BR}(\mu \rightarrow e \gamma)}{10^{-11}}}. \quad (4)$$

If this were the case, minimal SO(10) effects would not generate a detectable d_μ given the bounds on d_e (unless d_{ee} has a small CP-violating phase) and on $\mu \rightarrow e \gamma$. The most recent bounds are collected in table 1.

Realistic relations between lepton-slepton mixing and CKM matrix can only be obtained under assumptions on the lepton and down-quark Yukawa matrices. A $d_\mu \gtrsim 10^{-24} e \text{ cm}$ can arise if the $\mathcal{O}(3)$ Clebsh factors suppress the 12 and 21 entries of the lepton Yukawa matrix, so that $V_{e_L \tilde{\tau}_L}, V_{e_R \tilde{\tau}_R}$ are small. However, the approximate unification of the down and lepton Yukawa matrices, and the approximate equality $|V_{us}| \sim \sqrt{m_d/m_s}$ suggest a different assumption on the charged lepton Yukawa matrix [19]. It is interesting to consider the broad class of models with vanishing or sufficiently small 11, 13 and 31 entries of the charged lepton Yukawa matrix. One gets

$$\frac{d_e}{d_\mu} = \frac{d_{ee}}{d_{\mu\mu}} = -\frac{m_e}{m_\mu} \quad \text{and} \quad d_\mu = 2.4 \cdot 10^{-25} e \text{ cm} \sin \varphi_\mu \sqrt{\frac{\text{BR}(\mu \rightarrow e \gamma)}{10^{-11}}}. \quad (5)$$

¹This implies that different entries of the Yukawa matrices have different gauge structures, as given e.g. by Higgs fields in bigger representations of the unified gauge group, or by higher dimensional operators. These situations do not necessarily imply non universal A -terms: computable non universal RGE corrections [20] to the A -terms get canceled by computable GUT threshold effects [21], as dictated in a non trivial way by supersymmetry.

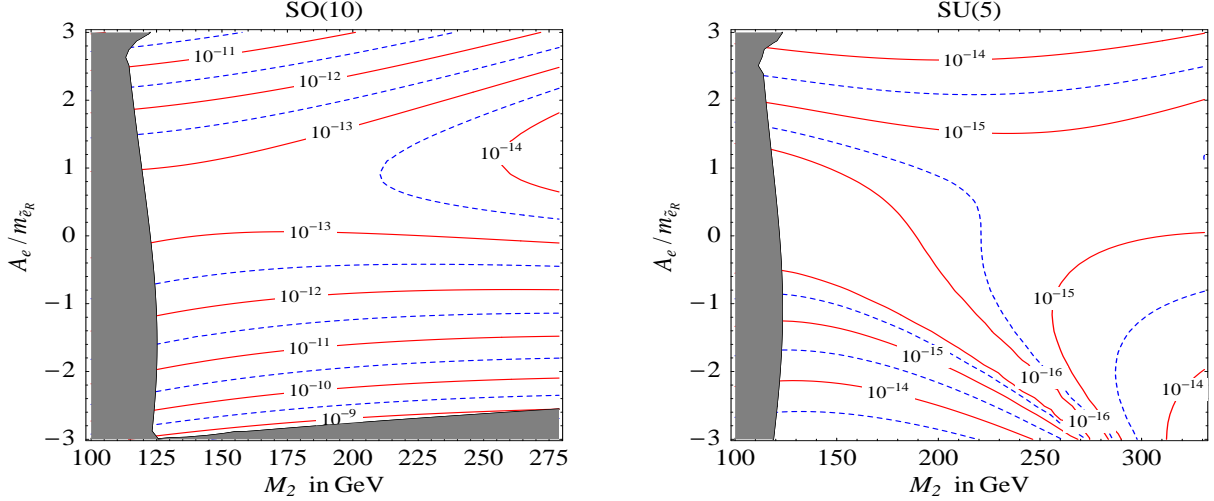


Figure 1: *Contour plot of $\text{BR}(\mu \rightarrow e\gamma)$ in SO(10) (left) and SU(5) (right), for lepton-slepton mixings as in eq. (3) and the parameter choice in eq. (7).*

where φ_μ is the phase of $d_{\mu\mu}$. Notice the opposite sign of d_μ/d_e with respect to the naïve scaling relation, as discussed in greater detail in section 5.

Unlike the ratio d_e/d_μ , the values of d_e , d_μ and of the $\mu \rightarrow e\gamma$ rate depend also on the structure of the 23 and 32 entries of the charged lepton and down quark Yukawa matrices. In fig. 1a we show the prediction for $\text{BR}(\mu \rightarrow e\gamma)$ in the minimal SO(10) model assuming eq. (3). The $\mu \rightarrow e\gamma$ rate obtained for any given assumption on the mixings, and the corresponding predictions for the muon and electron EDMs, can be then obtained by rescaling fig. 1a according to eq.s (1) and (2). We also show the prediction for $\text{BR}(\mu \rightarrow e\gamma)$ in the minimal SU(5) model in fig. 1b, although, as remarked above, SU(5) does not generate sizable d_μ and d_e . Both figures update the results obtained in [10]. In the rest of this section, we depart from the main theme of our work and discuss the details of the computation. The uninterested reader might want to jump to the next section.

An update of the computation in [10] is useful for various reasons. The main reason is that the effects we are considering strongly depend on the top Yukawa coupling at the GUT scale, see eq. (8). Before knowing the value of the top mass, $\lambda_t(M_{\text{GUT}})$ was estimated from theoretical prejudices about proximity to an infrared-fixed point and about exact bottom/tau unification [10]. There are other less relevant reasons to update the computation. Charged sparticles lighter than 100 GeV have been excluded by LEP. In the MSSM, a small $\tan\beta \lesssim 2$ gives a too light higgs. In the SU(5) case, the computation in [10] missed one diagram [22]: this gives an order one correction (the correct expression is given in appendix 2). Finally, assuming that the “ $g - 2$ anomaly” [23] is due to supersymmetry (rather than to QCD), it could be of some interest to study its implications for other sleptonic penguin effects, like $\mu \rightarrow e\gamma$ and the electric dipoles.

As in [10], we assume minimal SU(5) and SO(10) unification models, unified gaugino masses, universal scalar masses m_0 and universal trilinear real A -terms A_0 at $M_{\text{Pl}} = 2.4 \cdot 10^{18}$ GeV. The assumption of fully universal soft terms is not demanded by experimental or theoretical requirements, but is often employed in order to reduce the number of parameters (in our computation it fixes the value of the μ term and of other less relevant parameters).

RGE corrections in GUT models lead to non universal, flavor and CP-violating soft terms at the Fermi scale, inducing observable effects in leptons through one loop diagrams. In SO(10) the effects are dominated by electro-magnetic penguin diagrams. They dominate also in SU(5) if $\tan\beta \gtrsim \text{few}$. This

implies relations between different lepton flavor violating signals, like

$$\frac{\text{CR}(\mu \rightarrow e \text{ in Ti})}{\text{BR}(\mu \rightarrow e\gamma)} \approx 0.5 \cdot 10^{-2}, \quad \frac{\text{BR}(\mu \rightarrow e\bar{e}e)}{\text{BR}(\mu \rightarrow e\gamma)} \approx 0.7 \cdot 10^{-2}. \quad (6)$$

The correlation between $\mu \rightarrow e\gamma$ with d_e and d_μ (previously discussed) and with $\tau \rightarrow \mu\gamma$ (see [10]) is different in the SU(5) and SO(10) cases. The rates for all these other processes can be read from the $\mu \rightarrow e\gamma$ rate, that we plot in fig. 1a,b as function of $M_2, A_e/m_{\tilde{e}_R}$. We have assumed eq. (3) and

$$\lambda_t(M_{\text{GUT}}) = 0.6, \quad \tan \beta = 5, \quad m_{\tilde{e}_R} = 300 \text{ GeV}, \quad |V_{ts}| = 0.04, \quad |V_{td}| = 0.01, \quad \mu > 0 \quad (7)$$

Unless otherwise indicated all parameters are renormalized at the electroweak scale. For any other value of the parameters BR($\mu \rightarrow e\gamma$) can still be read from fig. 1 because it approximatively scales as

$$\text{BR}(\mu \rightarrow e\gamma) \propto \lambda_t^p(M_{\text{GUT}}) \times m_{\tilde{e}_R}^{-4} \times (\mu \tan \beta)^2 \times \begin{cases} |V_{e_R \tilde{\tau}_R} V_{\mu_R \tilde{\tau}_R}|^2 & \text{in SU(5)} \\ |V_{e_R \tilde{\tau}_R} V_{\mu_L \tilde{\tau}_L}|^2 + |V_{e_L \tilde{\tau}_L} V_{\mu_R \tilde{\tau}_R}|^2 & \text{in SO(10)} \end{cases} \quad (8)$$

where $p = 4$ in SU(5) and $p = 8$ in SO(10). When rescaling $m_{\tilde{e}_R}$ one has also to rescale the other mass parameters, M_2 and A_e . This naïve rescaling is a good approximation, unless sparticle masses are comparable to the Z mass. The upper bound on M_2 in fig. 1a,b corresponds to the assumption $m_0^2 > 0$. A large $\tan \beta$ can be naturally obtained from a small $\mu \propto 1/\tan \beta$, or by accidental cancellations. We have assumed a positive μ term, as suggested by data about $b \rightarrow s\gamma$ (and $g-2$, if attributed to supersymmetric effects), and computed it assuming an universal scalar mass term m_0 . For this choice of μ there can be accidental cancellations between the Feynman diagrams that contribute to the $\mu \rightarrow e\gamma$ rate in SU(5) (see fig. 1b).

In order to do a precise computation we fixed the value of $\lambda_t(M_{\text{GUT}})$, rather than the value of M_t . Fig. 2 (from [24]) shows the value of $\lambda_t(M_{\text{GUT}})$ extracted from the measured pole top mass

$$M_t = (175 \pm 5) \text{ GeV}.$$

The values of $\lambda_t(M_{\text{GUT}})$ are somewhat smaller than the ones used in [10], if values of $\tan \beta \lesssim 2$ are excluded because give a too light Higgs mass (as happens in the MSSM, unless one adds extra singlets). Decreasing $\lambda_t(M_{\text{GUT}})$ by a factor 2 reduces the SU(5) (SO(10)) prediction for the $\mu \rightarrow e\gamma$ decay rate by one (two) orders of magnitude. The dependence of the predicted $\mu \rightarrow e\gamma$ rate on $\tan \beta$ is not only due to the explicit $\tan^2 \beta$ factor in eq. (8), but also to the dependence of the experimental band for $\lambda_t(M_{\text{GUT}})$ on $\tan \beta$ depicted in fig. 2. As a consequence, the $\mu \rightarrow e\gamma$ rate is minimal around $\tan \beta \sim 4$.

The width of the range for $\lambda_t(M_{\text{GUT}})$ is not only due to the few % uncertainty on M_t , but also to unknown sparticle threshold corrections affecting the value of λ_t just above the SUSY breaking scale. Moreover, the running up to M_{GUT} depends on the gauge couplings (also affected by unknown sparticle threshold corrections) and amplifies the uncertainties in λ_t . In fact, the RGE evolution of λ_t exhibits an infra-red fixed point behavior: different values of $\lambda_t(M_{\text{GUT}})$ converge in a restricted range of values of λ_t at low energy [25]. This allowed to guess the value of M_t from assumptions about $\lambda_t(M_{\text{GUT}})$ [25]. Now that M_t is known we would like to do the opposite and renormalize λ_t from low to high energies. We then see the reverse of the medal: $\lambda_t(M_{\text{GUT}})$ is not strongly constrained by the measured value of M_t . Even assuming an exactly known M_t , sparticle threshold effects

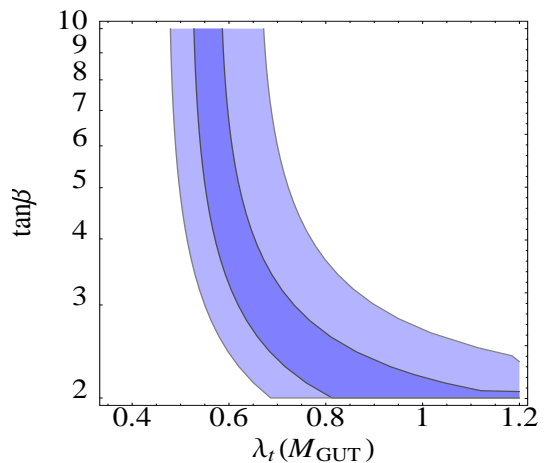


Figure 2: Values of $\lambda_t(M_{\text{GUT}})$ compatible with $M_t = (175 \pm 5) \text{ GeV}$ (outer band) and with $M_t = 175 \text{ GeV}$ (inner band).

would still induce a large uncertainty in $\lambda_t(M_{\text{GUT}})$ (as shown by the inner band in fig. 2). It is useful to remember that in the pure SM the theoretical error on the relation between M_t and $\lambda_t(M_t)$ (NNLO QCD corrections have not been computed) is equivalent to a ± 2 GeV uncertainty in M_t .

One can wonder if attributing the $g - 2$ anomaly to supersymmetry (rather than to QCD effects) would allow to constrain the predictions for the EDMs and $\mu \rightarrow e\gamma$ by reducing the uncertainty on the values of $\mu, \tan \beta$ and of the slepton masses. In fact the one loop diagrams that contribute to $\mu \rightarrow e\gamma$ and to $g - 2$ are quite similar. This analysis was done in [26] for the case discussed in the next section, but is particularly interesting in the case of the GUT-induced effects, where the uncertainty on sparticle masses induces the dominant uncertainty on the $\mu \rightarrow e\gamma$ rate. We find a strong correlation only if we assume that $m_0 = A_0 = 0$ at M_{Pl} ², in which case the A -terms and sfermion masses are induced by radiative corrections proportional to the gaugino masses. In this case we find

$$\text{BR}(\mu \rightarrow e\gamma) \approx \begin{cases} 3 \cdot 10^{-13} \lambda_t^4(M_{\text{GUT}}) \left(\frac{\delta a_\mu}{4 \cdot 10^{-9}} \right)^2 \left| \frac{V_{e_R \tilde{\tau}_R} V_{\mu_R \tilde{\tau}_R}}{0.01 \cdot 0.04} \right|^2 & \text{in SU(5)} \\ 2 \cdot 10^{-11} \lambda_t^8(M_{\text{GUT}}) \left(\frac{\delta a_\mu}{4 \cdot 10^{-9}} \right)^2 \frac{|V_{e_R \tilde{\tau}_R} V_{\mu_L \tilde{\tau}_L}|^2 + |V_{e_L \tilde{\tau}_L} V_{\mu_R \tilde{\tau}_R}|^2}{2(0.01 \cdot 0.04)^2} & \text{in SO(10)} \end{cases}, \quad (9)$$

where δa_μ represents the supersymmetric contribution to $a_\mu = (g - 2)/2$. In the general case, there is only a loose correlation between $\mu \rightarrow e\gamma$ and δa_μ . One reason is that the GUT-induced $\mu \rightarrow e\gamma$ rate depends on how much RGE effects due to the unified top Yukawa coupling make the staus lighter than selectrons and smuons: the amount of non-degeneracy depends on m_0 and A_0 and is minimal at $m_0 = A_0 = 0$. Therefore, in the SO(10) case, eq. (9) provides a lower bound on the $\mu \rightarrow e\gamma$ rate. In the SU(5) case there can be accidental cancellations between the Feynman diagrams that contribute to $\mu \rightarrow e\gamma$ (see fig. 1b): if $m_0, A_0 \neq 0$ the $\mu \rightarrow e\gamma$ rate can be above or below the value in (9).

3 Effects from neutrinos

Similar effects can be generated by the neutrino Yukawa couplings present in supersymmetric see-saw models. Adding to the MSSM “right-handed neutrinos” N_i , the most generic lepton superpotential

$$\mathcal{W} = \frac{M_{ij}}{2} N_i N_j + \lambda_N^{ij} L^i N^j H_u + \lambda_E^{ij} E^i L^j H_d, \quad (10)$$

gives the Majorana neutrino masses

$$\mathbf{m}_\nu = -\boldsymbol{\lambda}_N \cdot \frac{v^2 \sin^2 \beta}{M} \cdot \boldsymbol{\lambda}_N^T \quad (11)$$

(when appropriate we use boldface to emphasize the matrix structure). Through the same mechanism operative in GUT models, RGE effects proportional to the squared Yukawa coupling of the right-handed neutrinos imprint CP and lepton flavor violations in slepton masses. For example, the correction to the 3×3 mass matrix of left-handed sleptons is

$$\delta \mathbf{m}_{\tilde{L}}^2 = -\frac{1}{(4\pi)^2} (3m_0^2 + A_0^2) \mathbf{Y}_N + \dots, \quad \text{where} \quad \mathbf{Y}_N \equiv \boldsymbol{\lambda}_N^* \ln\left(\frac{M_{\text{GUT}}^2}{M M^\dagger}\right) \boldsymbol{\lambda}_N^T \quad (12)$$

having assumed universal soft terms at M_{GUT} and neglected $\mathcal{O}(\lambda_N^4)$ effects. In this approximation, the experimental bounds from $\ell_i \rightarrow \ell_j \gamma$ decays are saturated for

$$[\mathbf{Y}_N]_{\tau\mu}, [\mathbf{Y}_N]_{\tau e} \sim 10^{1\pm 1}, \quad [\mathbf{Y}_N]_{\mu e} \sim 10^{-1\pm 1} \quad (13)$$

²This ‘no scale’ [27] boundary condition is probably the simplest way of justifying universal sfermion masses.

having assumed values of sparticle masses compatible with a supersymmetric explanation of the $g - 2$ anomaly. A more complete analysis can be found in [28].

Since \mathbf{M} is unknown, the see-saw relation (10) does not allow to convert the measurement of the neutrino masses \mathbf{m}_ν into useful restrictions on the scale of λ_N , or on its flavor structure. In particular, the large ν_μ/ν_τ mixing observed in atmospheric oscillations [29] does not necessarily imply a correspondingly large SUSY mixing in slepton interactions. The \mathbf{M} , λ_N and λ_E matrices that describe the supersymmetric see-saw superpotential of eq. (10) contain 15 real parameters and 6 CP-violating phases. At low energy, in the mass eigenstate basis of the leptons, 3 real parameters describe the lepton masses, and both the neutrino and the left-handed slepton mass matrices are described by 6 real parameters and 3 CP-violating phase [30]. Since $(15 + 6) = (3 + 0) + (6 + 3) + (6 + 3)$ we see that see-saw mechanism has too many free parameters to allow to make general predictions: *any pattern of lepton and neutrino masses is compatible with any pattern of radiatively-generated flavor violations in left-handed slepton masses*³. The RGE effects in A -terms can be predicted in terms of the RGE effects in left-handed slepton masses. Unlike in the GUT case, it is not possible to predict the relative size of the different flavor and CP violating processes like $\mu \rightarrow e\gamma$, $\tau \rightarrow \mu\gamma$, d_e , d_μ and to assess which one has a better chance of being observed, if any. For these reasons we will not perform ‘detailed’ computations.

In view of this situation, one can try to see if useful informations can be obtained from models of fermion masses, rather than directly from fermion masses. In this respect, it is important to appreciate that the requirement of having a large atmospheric mixing angle between the *most splitted* neutrino states ($\Delta m_{\text{atm}}^2 \gg \Delta m_{\text{sun}}^2$) gives significant restrictions⁴, suggesting two peculiar structures for the Majorana neutrino mass matrix \mathbf{m}_ν : in the limit $\Delta m_{\text{sun}}^2 = 0$ they are

- (a) a rank one matrix (if $\Delta m_{\text{atm}}^2 > 0$, i.e. if neutrinos have a hierarchical spectrum);
- (b) a rank two pseudo-Dirac matrix (if $\Delta m_{\text{atm}}^2 < 0$, i.e. if neutrinos have an inverted spectrum)⁵.

Assuming $\theta_{\text{atm}} = \pi/4$ and $\theta_{\text{CHOOZ}} = 0$ (i.e. maximal $\nu_\mu \leftrightarrow \nu_\tau$ atmospheric oscillations), in the mass eigenstate basis of charged leptons, these matrices can be explicitly written as

$$\mathbf{m}_\nu(\text{a}) \propto \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}, \quad \mathbf{m}_\nu(\text{b}) \propto \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$

In the see-saw context, these mass matrices are generated by the following superpotentials [34, 35]

$$\mathcal{W}(\text{a}) = \lambda N(L_\mu + L_\tau)H_u + \frac{M}{2}N^2, \quad \mathcal{W}(\text{b}) = \lambda N(L_\mu + L_\tau)H_u + \lambda' N' L_e H_u + M N N' \quad (14)$$

that could be both justified by a broken $L_e - L_\mu - L_\tau$ symmetry (that would suppress $\mu \rightarrow e\gamma$). Having reproduced the main structure of the neutrino mass matrix, it is easy to add the solar mass splitting (large solar mixing is automatically obtained in case b), and to inglobate neutrino masses in a full model

³Since there are no restrictions, it is of course possible to obtain values of $\text{BR}(\mu \rightarrow e\gamma)$ or of $\text{BR}(\tau \rightarrow \mu\gamma)$ just below their experimental bounds. This is necessarily the case, under appropriate assumption on the size of the neutrino Yukawa couplings [31, 26] eventually justified by choosing some flavor model [32]. On the contrary, too large $\mu \rightarrow e\gamma$ rates are normally obtained under other reasonable assumptions not demanded by experimental data, as discussed in [33]. In general, eq.s (13) tell that a significant improvement of the experimental sensitivity in $\tau \rightarrow \mu\gamma$ would allow to test if the neutrino Yukawa matrix contains order one entries.

⁴Experimental data prefer, but do not require, $\Delta m_{\text{sun}}^2 \ll \Delta m_{\text{atm}}^2$. If this were not the case, neutrino data would not give significant restrictions on flavor models. Furthermore we restrict our attention on models that predict $\theta_{\text{atm}} \sim 1$. Depending on personal taste, one could instead be content with models that predict $\theta_{\text{atm}} \sim \epsilon^{1/2}$ (where ϵ is a relatively small number), or want models that predict θ_{atm} close to $\pi/4$.

⁵Neutrinos could also have a degenerate or quasi-degenerate spectrum, but we are not interested in these cases. We also do not consider inverted spectra with a generic Majorana phase incompatible with a pseudo-Dirac structure, that automatically guarantees the smallness of $\Delta m_{\text{sun}}^2/\Delta m_{\text{atm}}^2$ and makes it stable under radiative corrections.

of fermion masses. In conclusion, the only generic suggestion from neutrino data is that one right handed neutrino mass eigenstate has comparable Yukawa couplings to τ and μ and a smaller coupling to e . What are the implications of this restricted structure for supersymmetric lepton-flavor and CP-violating effects? The Yukawa coupling λ is the crucial parameter for supersymmetric effects, but its size cannot be deduced from the see-saw relation in eq. (11). Concrete examples that illustrate how both large and small SUSY mixings are compatible with this restricted see-saw structure have been presented in [28] (for case (a) — the extension to case (b) is trivial). In fact, the see-saw structure (14) does not need large mixing angles in any Yukawa matrix (if λ is not the largest element of λ_N : see [36] for explicit examples). The casistics discussed in [28] contains all what can be reliably said about neutrino-Yukawa-induced $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ decays.

In the assumption that λ is large enough (e.g. $\lambda \sim \lambda_t$), the minimal see-saw structure of eq. (14) generates significant effects in the τ, μ sector (e.g. the $\tau \rightarrow \mu\gamma$ decay, see eq. (13)) but does not generate a muon electric dipole. In fact, by redefining the phases of left and right-handed leptons, CP violation can be rotated away from this large Yukawa coupling λ and confined to the smaller Yukawa couplings of the other right-handed neutrinos.

Is it possible to obtain sizable lepton EDMs by choosing appropriate values for the many unknown see-saw parameters (compatible with neutrino masses, but not suggested by them)? We need to further assume that many entries of λ_N are large, with the hierarchy in neutrino masses obtained from a hierarchy in the masses M . The computation of electric dipole moments is tricky, but general arguments simplify in a useful way the analysis. (We remind that here we assume that soft terms are universal at M_{GUT} : the more interesting case of unified see-saw models with universal soft terms at M_{Pl} will be considered in the next section). Then the 3×3 flavor matrix of charged lepton electro-magnetic moments \mathbf{d} (see eq.s (1) or (26)) is restricted by the $U(3)_L \otimes U(3)_E \otimes U(3)_N$ symmetry of the flavor-universal part of the Lagrangian and by the holomorphicity of supersymmetry. The contribution to \mathbf{d} that dominates the EDMs has the form⁶

$$\mathbf{d} \propto \lambda_E \cdot \mathbf{Y}_N \cdot \mathbf{Y}_E \cdot \mathbf{Y}_N^2 \quad \text{where} \quad \mathbf{Y}_E \equiv \lambda_E^\dagger \cdot \lambda_E \quad (15)$$

and \mathbf{Y}_N is defined in eq. (12). The muon electric dipole is the imaginary part of the $\mu\mu$ diagonal element of the matrix \mathbf{d} , as in eq. (1). Eq. (15) gives electric dipoles d_e and d_μ suppressed by λ_τ^2 , so that large dipoles can be obtained at large $\tan\beta$:

$$d_\ell \sim \frac{e\alpha_{\text{em}}}{4\pi} \frac{m_\ell}{\tilde{m}_{\tilde{\ell}}^2} \lambda_{N_3}^4 \lambda_{N_2}^2 \lambda_\tau^2 J'_{\text{CP}} \quad (16)$$

where $\lambda_{N_3}, \lambda_{N_2}$ are the largest and next-to-largest neutrino Yukawa eigenvalues and J'_{CP} is the Jaroskog invariant associated to the matrices \mathbf{Y}_N and λ_E . If $\lambda_{\nu_3} \sim \lambda_{\nu_2} \sim 1$ large lepton EDMs can be generated. However, whatever is the flavor structure of λ_N , the electric dipoles satisfy the relation $d_\mu/m_\mu \approx -d_e/m_e$ (as can be understood by noticing that d_τ/m_τ and $\sum_\ell d_\ell/m_\ell \propto \text{Im Tr}(\mathbf{Y}_N \cdot \mathbf{Y}_E \cdot \mathbf{Y}_N^2)$ are small [14]). The naïve scaling between d_e and d_μ can be evaded using the flavor structures that only appear at subleading orders (e.g. two loop RGE), different from \mathbf{Y}_N because contain a smaller power of $\ln(M_{\text{GUT}}^2/M M^\dagger)$. Since large neutrino couplings λ_N^{ij} imply M_{ij} not much smaller than M_{GUT} , the d_μ generated by ‘sub-leading’ radiative effects could be sizable and larger than the one generated by ‘leading’ radiative effects. The most optimistic thing that can be said about this possibility is that it is not excluded.

4 Effects from neutrinos and from SU(5)

A more appealing possibility arises if the gauge structure at the scale at which the soft terms are universal is richer than the SM one. As seen in section 2, an enhancement of LFV processes and EDMs requires

⁶For brevity, here we only give the final result. The use of $U(3)$ symmetries is explained in [14]. The consequences of the holomorphicity of supersymmetry for soft breaking terms have been nicely described in [37]. Alternatively, the way in which the factors λ_N and M appear can be understood by inspecting the explicit form of the RGE equations.

non-universality in both the right-handed and left-handed slepton sectors. RGE corrections due to right handed neutrinos do not affect right handed sleptons, and RGE effects do to the unified Yukawa of the top in SU(5) GUTs do not affect left-handed sleptons. If both neutrino and SU(5) effects are present, the amplitudes of penguin diagrams (that give EDMs and $\ell_i \rightarrow \ell_j \gamma$ decays) get enhanced by m_τ/m_μ factors as in SO(10). The relevant Yukawa interactions are described by the superpotential

$$\mathcal{W} = \lambda_U^{ij} T_i T_j H + \lambda_{DE}^{ij} F_i T_j \bar{H} + \lambda_N^{ij} F_i N_j H$$

where T (F) are the usual SU(5) 10-plets ($\bar{5}$ -plets) and H, \bar{H} are the SU(5) Higgs fields. It is convenient to assume that \mathcal{W} is written in the mass-eigenstate basis of the right-handed neutrino singlets N_j . As discussed in the previous section, the atmospheric neutrino anomaly and the CHOOZ bound motivate the assumption that one of the right-handed neutrinos (say, N_3) has comparable couplings to the second and third generation and a smaller coupling to the first generation:

$$\lambda_N^{i3} \equiv U_{i3} \lambda, \quad \text{with} \quad U_{\tau 3} \sim U_{\mu 3} \gg U_{e3} \quad \text{and} \quad U_{e3}^2 + U_{\mu 3}^2 + U_{\tau 3}^2 = 1.$$

We now show that a large d_μ can be obtained in this minimal unified see-saw context, provided that $\lambda \approx \sqrt{M/10^{15} \text{ GeV}}$ is large enough. As previously discussed, this requires to assume an appropriate value, around the GUT scale, for the mass M of the right-handed neutrino N_3 .

Since we are interested in the electric dipoles of the charged leptons we can neglect the small up and charm couplings in the up-quark matrix, $\lambda_c = \lambda_u = 0$. In this unified see-saw context we can also conservatively assume that the other neutrino Yukawa couplings are small and neglect them: $\lambda_N^{i2} = \lambda_N^{i1} = 0$. The important point is that in this relevant limit it is no longer possible to rotate away CP-violation from the relevant Yukawa interactions

$$\mathcal{W} = \lambda_t T_3 T_3 H + \lambda_{DE}^{i3} F_i T_3 \bar{H} + \lambda_N^{i3} F_i N_3 H.$$

The phases in λ_t , in λ_{DE}^{i3} and one of the phases in λ_N^{i3} can be rotated away⁷ by redefining the phases in T_3, F_i, N_3 , but the remaining phases in λ_N^{i3} are physical. They induce the following EDMs

$$d_e \sim \frac{\alpha_{\text{em}} m_\tau}{4\pi m_\tau^2} \lambda^2 \lambda_t^2 \text{Im } V_{td} U_{e3} (U_{\tau 3} V_{tb})^*, \quad d_\mu \sim \frac{\alpha_{\text{em}} m_\tau}{4\pi m_\tau^2} \lambda^2 \lambda_t^2 \text{Im } V_{ts} U_{\mu 3} (U_{\tau 3} V_{tb})^*,$$

where we have assumed eq.s (3). Therefore, d_μ can be large and much larger than d_e . Using the techniques described in the previous section, the same result can be reobtained in a more elegant way by noticing that the contribution to \mathbf{d} relevant for the EDMs is proportional to $\lambda_U \lambda_U^\dagger \lambda_{DE}^T \lambda_N^* \lambda_N^T$. We now discuss the bounds on d_μ set by other processes. The strongest bound comes from $\tau \rightarrow \mu \gamma$ and depends on the details of the model. Omitting unknown order one factors and assuming large CP-violating phases we estimate⁸

$$d_\mu \sim \text{few} \cdot 10^{-23} \text{ e cm} \frac{|V_{ts}|}{0.04} \sqrt{\frac{\text{BR}(\tau \rightarrow \mu \gamma)}{10^{-6}}}.$$

Since $d_\mu \lesssim d_e V_{ts}/V_{td} U_{e3}$ the experimental bound on d_e induce an upper bounds on d_μ . The upper bound on d_μ induced by the experimental bound on $\mu \rightarrow e \gamma$ is comparable and also dependent on the unknown parameter U_{e3} . As remarked above, the parameter U_{e3} is not directly related to the corresponding measurable element of the neutrino mixing matrix: U_{e3} can be naturally smaller than θ_{CHOOZ} (that can be generated by another right-handed neutrino). The opposite possibility, $\theta_{\text{CHOOZ}} \ll U_{e3}$, would instead need accidental cancellations. Within the minimal see-saw structure suggested by neutrino data, eq. (14), the additional assumption that the lepton Yukawa matrix has a vanishing 11 element and comparable 12 and 21 entries gives rise to a relatively large $U_{e3} \sim \sqrt{m_e/2m_\mu}$ (that will be experimentally tested in long-baseline neutrino experiments) and consequently to $d_\mu/m_\mu \sim d_e/m_e$.

⁷In order to rotate away one of the phases in λ_N^{i3} one has to redefine the N_3 field, eventually giving a complex right-handed neutrino mass term M . However, the effects we are considering only depend on $M^\dagger M$, so that a complex M is indeed irrelevant.

⁸Bounds from $\tau \rightarrow \mu \gamma$ have been discussed in [13] in a more general context but using the ‘mass-insertion approximation’. An analogous analysis has been performed in [38] in the quark sector.

5 Flavor symmetries

We now consider the possibility that the structure of soft terms is constrained by the same physics giving rise to the pattern of fermion masses and mixings. A generic flavor symmetry that explains the observed pattern of fermion masses does not necessarily force an acceptable pattern of sfermion masses. For example, $U(1)$ symmetries do not relate the diagonal soft mass terms, all separately invariant under the symmetry. On the other hand, the required approximate degeneracy of the first and second family sfermion masses can be guaranteed by a suitable non-abelian symmetry. It is then interesting that the simplest non-abelian symmetry accounting for the smallness of the two lighter fermion families masses also automatically guarantees this approximate degeneracy [39]. In this section, we discuss the expectations for the muon and electron EDMs in this context.

Rather than focusing on a particular model, we would like to study the generic properties of this class of models. In order to simplify the discussion we will make few assumptions, valid in wide classes of models. Generically, the lepton Yukawa matrix and the slepton masses can be diagonalized by unitary matrices U_L, U_R, T_L, T_R as

$$\lambda_E = U_R^\dagger \lambda_{E,\text{diag}} U_L, \quad m_L^2 = T_L^\dagger m_{L,\text{diag}}^2 T_L, \quad m_{e_R}^2 = T_R^\dagger m_{e_R,\text{diag}}^2 T_R. \quad (17)$$

For a naturally hierarchical Yukawa matrix λ_E , the mixing matrices $U_{L,R}$ can be written with sufficient accuracy as the product of three 2×2 rotations diagonalizing sector by sector the Yukawa matrix in subsequent steps: $U_L = U_{13}^L U_{12}^L U_{23}^L$, $U_R = \Phi U_{31}^R U_{21}^R U_{32}^R$, where $\Phi = \text{diag}(e^{i\phi_e}, e^{i\phi_\mu}, e^{i\phi_\tau})$ removes the phases from the diagonal elements one is left with after the three rotations. Since the Yukawa matrices are in general complex, the “ ij ” rotations also involve a phase in their off diagonal elements.

The first assumption we make is that the 13 rotations are negligible. This is the case if the 13 and 31 entries of the charged lepton Yukawa matrix are sufficiently small. In the quark sector, the same assumption on the quark Yukawa matrices is supported by the pattern of quark masses and mixings. In fact, the presence of approximate “texture zeros” in the 13, 31 positions, together with the presence of an approximate zero in the 11 position and the approximate equality in magnitude of the 12 and 21 elements, successfully accounts for the “leading order” [40] relations between quark masses and angles $|V_{ub}/V_{cb}| \sim \sqrt{m_u/m_c}$, $|V_{td}/V_{ts}| \sim \sqrt{m_d/m_s}$ [41]. Here, however, we will only make the assumption that the 13 and 31 elements of λ_E are negligible. Since the Yukawa matrix and the corresponding A -term matrix have the same quantum numbers under the flavor symmetry determining their structure, the A -terms can be written as $\tilde{A}_{ij} = A_{ij} \lambda_{ij}$, where the various A_{ij} are naturally comparable but not equal.

Secondly, we assume that effects due to non degeneracy between selectrons and smuons can be neglected. Any successfully broken flavor symmetry must guarantee that degeneracy at a high level of accuracy. Finally, we again assume that 13 rotations can be neglected also in the slepton sectors. In conclusion, the left and right-handed slepton mass matrices can be diagonalized by a rotation in the “23 sector” only. If the rotation in the 23 sector is not too large, this hypothesis takes under control potentially dangerous FCNC and CP-violation effects.

To summarize, we assume the following structures

$$\lambda = \begin{pmatrix} \lambda_{11} & \lambda_{12} & 0 \\ \lambda_{21} & \lambda_{22} & \lambda_{23} \\ 0 & \lambda_{32} & \lambda_{33} \end{pmatrix}, \quad \tilde{A} = A\lambda = \begin{pmatrix} A_{11}\lambda_{11} & A_{12}\lambda_{12} & 0 \\ A_{21}\lambda_{21} & A_{22}\lambda_{22} & A_{23}\lambda_{23} \\ 0 & A_{32}\lambda_{32} & A_{33}\lambda_{33} \end{pmatrix}, \quad m^2 = \begin{pmatrix} m_{1,2}^2 & 0 & 0 \\ 0 & m_{1,2}^2 & m_{23}^2 \\ 0 & m_{23}^2 & m_3^2 \end{pmatrix} \quad (18)$$

for the various Yukawa, A -term⁹ and sfermion mass matrices.

We are now ready to compute the EDMs¹⁰. Let us first consider the muon EDM. As discussed in appendix A, eq.s (28,29), the general one-loop expression for d_μ , eq. (27), can be conveniently simplified as

⁹A similar structure of A -terms has been considered in [42].

¹⁰The supersymmetric flavor problem can be partially alleviated by alternatively assuming that the sfermions of the first two generations have few TeV mass. This possibility would give similar EDMs as the one we consider here.

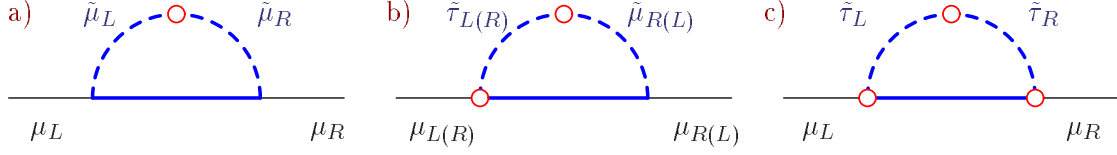


Figure 3: The three type of contributions to d_μ . The photon (not shown) should be coupled to charged particles.

a sum of three dominant contributions, pictorially represented in fig. 3. The first diagram is proportional to $\tilde{A}_{\mu_L\mu_R}$, the 22 element of the A -term matrix, written in the lepton mass eigenstate basis (i.e. the muon A -term). The last diagram employs flavor violation at μ_L and μ_R vertices and is proportional to $V_{\mu_R\tilde{\tau}_R}\tilde{A}_{\tilde{\tau}_R\tilde{\tau}_L}V_{\tilde{\tau}_L\mu_L}^\dagger$, where $\tilde{A}_{\tilde{\tau}_R\tilde{\tau}_L}$ is the 33 element of the A -term matrix, written in the mass eigenstate basis of sleptons, and $V \sim UT^\dagger$ are the lepton/slepton mixings (precisely defined in appendix A). The intermediate diagrams are proportional to $V_{\mu_R\tilde{\tau}_R}\tilde{A}_{\tilde{\tau}_R\mu_L}$ and $\tilde{A}_{\mu_R\tilde{\tau}_L}V_{\tilde{\tau}_L\mu_L}^\dagger$, that involve the A -term matrix in the lepton-slepton mixed basis (their general expressions in terms of the U_L, U_R, T_L, T_R matrices can be found in appendix A, eq. (30)). Similar expressions hold for d_e . We now compute these flavor factors.

We begin with diagonalizing explicitly the charged lepton mass matrix. The values of $\lambda_e, \lambda_\mu, \lambda_\tau > 0$ are related to the complex elements of the lepton Yukawa matrix λ_{ij} by

$$\lambda_\tau e^{i\phi_\tau} = \lambda_{33}, \quad \lambda_\mu e^{i\phi_\mu} = \lambda_{22} - \epsilon_{23}\epsilon_{32}\lambda_\tau e^{i\phi_\tau}, \quad \lambda_e e^{i\phi_e} = \lambda_{11} - \epsilon_{12}\epsilon_{21}\lambda_\mu e^{i\phi_\mu} \quad (19)$$

where

$$\epsilon_{23} = \frac{\lambda_{23}}{\lambda_{33}}, \quad \epsilon_{32} = \frac{\lambda_{32}}{\lambda_{33}}, \quad \epsilon_{12} = \frac{\lambda_{12}}{\lambda_\mu e^{i\phi_\mu}}, \quad \epsilon_{21} = \frac{\lambda_{21}}{\lambda_\mu e^{i\phi_\mu}}$$

are the complex ij -rotation angles in $U_{32}^R, U_{23}^L, U_{21}^R, U_{12}^L$ respectively. Our equations hold if $|\epsilon_{ij}| \ll 1$, otherwise they are still qualitatively correct. In the standard basis where $\lambda_E = \text{diag}(\lambda_e, \lambda_\mu, \lambda_\tau)$, the lepton A -term matrix, that is the factor entering in the first contribution to d_e, d_μ (fig. 3a), is

$$U_R \tilde{A}_E U_L^\dagger = \begin{pmatrix} \lambda_e A_e & \epsilon_{12}\lambda_\mu e^{i(\phi_\mu - \phi_e)}(A_{12} - A_\mu) & -\epsilon_{12}\epsilon_{23}\lambda_\tau e^{i(\phi_\tau - \phi_e)}(A_{23} - A_{33}) \\ \epsilon_{21}\lambda_\mu(A_{21} - A_\mu) & \lambda_\mu A_\mu & \epsilon_{23}\lambda_\tau e^{i(\phi_\tau - \phi_\mu)}(A_{23} - A_{33}) \\ -\epsilon_{21}\epsilon_{32}\lambda_\tau(A_{32} - A_{33}) & \epsilon_{32}\lambda_\tau(A_{32} - A_{33}) & \lambda_\tau A_{33} \end{pmatrix}, \quad (20)$$

where

$$A_\mu = A_{22} + a_\mu(A_{22} - A_{23} - A_{32} + A_{33}), \quad a_\mu \equiv \epsilon_{23}\epsilon_{32}\frac{m_\tau}{m_\mu}e^{i(\phi_\tau - \phi_\mu)}, \quad (21a)$$

$$A_e = A_{11} + a_e(A_{11} - A_{21} - A_{12} + A_\mu), \quad a_e \equiv \epsilon_{12}\epsilon_{21}\frac{m_\mu}{m_e}e^{i(\phi_\mu - \phi_e)}. \quad (21b)$$

The quantities $\tilde{A}_{\mu_R\mu_L}, \tilde{A}_{e_R e_L}$ entering the first contribution to the EDMs (fig. 3a) are given respectively by $\lambda_\mu A_\mu$ and $\lambda_e A_e$. The coefficients a_e and a_μ are texture-dependent complex numbers, that measure how significant are the off-diagonal contributions to the e and μ mass, respectively. Their natural range is $0 < |a_\ell| < \text{few}$. In particular, one has $a_e = -1$ for a texture with a vanishing 11 element. The position of phases in eq. (20) reflects the choice of including the matrix of phases Φ in the right-handed rotation.

The two order one contributions (fig. 3b) need the elements of the A -term matrix in the mixed basis in which one $\tilde{\tau}$ is involved:

$$\begin{aligned} \tilde{A}_{\tilde{\tau}_R\mu_L} &= \epsilon_{32}\lambda_\tau(A_{32} - A_{33})e^{i\phi_\tau}, & \tilde{A}_{\tilde{\tau}_R e_L} &= -\epsilon_{21}\tilde{A}_{\tilde{\tau}_R\mu_L}, \\ \tilde{A}_{\mu_R\tilde{\tau}_L} &= \epsilon_{23}\lambda_\tau(A_{23} - A_{33})e^{i(\phi_\tau - \phi_\mu)}, & \tilde{A}_{e_R\tilde{\tau}_L} &= -\epsilon_{12}\tilde{A}_{\mu_R\tilde{\tau}_L}. \end{aligned} \quad (22)$$

Finally, the order two contribution (fig. 3c) involve the 33 element of the A -term matrix in the slepton mass basis, $\tilde{A}_{\tilde{\tau}_R \tilde{\tau}_L}$, which essentially coincides with $\lambda_\tau A_{33}$.

From the above equations we can finally recover the expression for the quantities in eq. (29) directly related to the EDMs. For the muon EDM we get

$$A_0^\mu = m_\mu A_\mu \quad (23a)$$

$$A_{1L}^\mu = e^{i(\phi_\tau - \phi_\mu)} \epsilon_{32} (\epsilon_{23} - \tilde{\epsilon}_{23}) m_\tau (A_{33} - A_{32}) \quad (23b)$$

$$A_{1R}^\mu = e^{i(\phi_\tau - \phi_\mu)} \epsilon_{23} (\epsilon_{32} - \tilde{\epsilon}_{32}) m_\tau (A_{33} - A_{23}) \quad (23c)$$

$$A_2^\mu = e^{i(\phi_\tau - \phi_\mu)} (\epsilon_{23} - \tilde{\epsilon}_{23}) (\epsilon_{32} - \tilde{\epsilon}_{32}) m_\tau (A_{33} + \mu \tan \beta), \quad (23d)$$

where $\tilde{\epsilon}_{32}$ ($\tilde{\epsilon}_{23}$) represent the contribution of the 23 rotation that diagonalizes the left-handed (right-handed) sfermion mass matrix. For the electron EDM we get

$$A_0^e = m_e A_e, \quad A_{1L}^e = A_{1L}^\mu a_e m_e / m_\mu, \quad A_{1R}^e = A_{1R}^\mu a_e m_e / m_\mu, \quad A_2^e = A_2^\mu a_e m_e / m_\mu. \quad (24)$$

We can now discuss the size of the different contributions to the EDMs. If the A -terms are complex, the trivial term due to A_e and A_μ (contained in A_0) would give $|d_e/d_\mu| \sim m_e/m_\mu$ for any texture of the lepton Yukawa matrix. A significant enhancement of d_μ/d_e would be possible only as a result of an accidental cancellation in the expression for d_e . The higher order contributions to the EDMs, $A_i^{\epsilon, \mu}$ with $i = \{1L, 1R, 2\}$, satisfy $\text{Im}(A_i^e/m_e) = \text{Im}(a_e A_i^\mu/m_\mu)$ and therefore would give $|d_e/d_\mu| \approx |a_e| m_e/m_\mu$. A sufficiently large d_μ is obtained if $|a_e| \ll 1$ (i.e. with an appropriate texture for λ_E) and if the $A_{1L,R}, A_2$ contributions are the dominant ones. The A_2 contribution dominates if $\tan \beta$ is large, or if $m_{\tilde{e}, \tilde{\mu}} \gg m_{\tilde{\tau}}$. On the contrary, these contributions are suppressed if $|a_\mu|$ (and the $\tilde{\epsilon}_{23}$ rotations) are small. If the large mixing angle observed in atmospheric neutrino oscillations comes from the charged lepton Yukawa matrix (i.e. if $|\epsilon_{23}| \sim 1$), we expect $a_\mu \sim 1$. In this case it is not possible to neglect 13 rotations [43] and one has a significant $\tau \rightarrow \mu \gamma$ rate.

Since strong bounds exist on the phase of some of the soft terms (e.g. the μ term) for reasonable sparticle masses [5], it is more appealing to consider the case of real (but non-universal) soft terms. CP violation is contained in the non diagonal Yukawa matrices, that can be diagonalized by flavor rotations with complex mixing angles ϵ_{ij} , as in (17). In general λ_E contains 4 independent phases, reduced to the 2 phases of a_e and a_μ , under our assumption that the 13 and 31 elements of the Yukawa matrix are negligible. In the physical basis where $\lambda_E = \text{diag}(\lambda_e, \lambda_\mu, \lambda_\tau)$, the slepton soft-terms are no longer real, giving rise to sizable EDMs as explicitly shown by eq.s (23,24). The phases of A_e, A_μ are naturally small since $\text{Im} a_\ell \ll 1$ unless the two contributions to λ_e, λ_μ in eq. (19) are comparable. With real A -terms, the zeroth order contributions A_0 gives

$$\text{Im} A_0^e = m_e (A_{33} - A_{23} - A_{32} + A_{22}) \text{Im} a_e a_\mu + m_e (A_{22} - A_{12} - A_{21} + A_{11}) \text{Im} a_e \quad (25a)$$

$$\text{Im} A_0^\mu = m_\mu (A_{33} - A_{23} - A_{32} + A_{22}) \text{Im} a_\mu. \quad (25b)$$

The higher order contributions to d_e are also proportional to a_e , so that a d_μ/d_e larger than m_μ/m_e is always obtained if $|a_e| \ll 1$ (i.e. if the electron mass is dominantly due to the 11 element of λ_E). A small $|a_e|$ could be due e.g. to a broken $L_e - L_\mu - L_\tau$ symmetry [35] motivated by neutrino data (as recalled in section 3). We remark, however, that $|a_e| \ll 1$ corresponds to a non vanishing λ_{11} dominating the electron mass. In the quark sector, the analogous situation would be incompatible with the measured value of V_{us} . It is interesting to study also the opposite situation of an approximate texture zero in the 11 element of λ_E , so that $a_e = -1$: in this case one always has $d_\mu/m_\mu = -d_e/m_e$ (rather than $d_\mu/m_\mu = +d_e/m_e$). Therefore, if the electron EDM and supersymmetry were discovered, a measurement of d_μ/d_e (and in particular of its sign) would be interesting, since it contains informations on the origin of the EDMs and, eventually, on the structure of the lepton Yukawa matrix. In the quark sector the analogous measurement of d_s/d_d could be performed if hadronic uncertainties [44] could be kept sufficiently under control and if d_u and θ_{QCD} are small enough.

We can finally pass to numerical results. How large can be d_μ , compatibly with the most recent bounds on related processes collected in table 1? It is convenient to translate the bounds on rare τ and μ decays into bounds on the transition dipoles $d_{\ell\ell'}$, defined by the effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \frac{1}{2} \sum_{\ell\ell'} [\bar{\ell}_R \gamma_{\mu\nu} d_{\ell\ell'} \ell'_L + \bar{\ell}'_L \gamma_{\mu\nu} d_{\ell\ell'}^* \ell_R] F^{\mu\nu} \quad (26)$$

where $\ell, \ell' = \{e, \mu, \tau\}$ and $\ell_{R(L)} = \frac{1}{2}(1 \pm \gamma_5)\ell$. When $m_\ell > m_{\ell'}$ the dipoles induce the decay $\Gamma(\ell \rightarrow \ell'\gamma) = m_\ell^3(|d_{\ell\ell'}|^2 + |d_{\ell'\ell}|^2)/(16\pi)$. For $\ell = \ell'$ they induce the electric dipole $d_\ell = \text{Im } d_{\ell\ell}$ and the magnetic moment $a_\ell = (2m_\ell/e)\text{Re } d_{\ell\ell}$. Assuming equal left and right-handed mixing one has

$$|d_{e\mu}| = |d_{\mu e}| < 2 \cdot 10^{-26} e \text{ cm}, \quad |d_{\tau\mu}| = |d_{\mu\tau}| < 5 \cdot 10^{-22} e \text{ cm}.$$

In view of the many unknown parameters (the sparticle masses, their mixings, ...) at the moment we do not find useful to present detailed numerical results. The dipoles are roughly given by

$$d_\ell \sim \frac{e\alpha_{\text{em}} m_\ell A_\ell}{24\pi \cos^2 \theta_W m_{\tilde{\ell}}^3} \text{Im } a_\ell, \quad d_{\mu\tau} \sim d_{\tau\mu} \sim d_\mu \sqrt{\frac{m_\tau}{a_\mu m_\mu}}, \quad d_{\mu e} \sim d_{e\mu} \sim d_\mu \sqrt{\frac{a_e m_e}{m_\mu}}$$

where A_ℓ represent the combinations of μ and A terms computed above (eventually enhanced by $\tan \beta$) and a_e, a_μ are the most relevant texture-dependent free parameters. The numerical factor is the one appropriate for the bino contribution to the zeroth-order term A_0^ℓ , evaluated for $m_{\tilde{\ell}} = m_{\tilde{B}}$. For $A_\mu = m_{\tilde{\mu}} = 100 \text{ GeV}$ and $a_\mu \sim 1$ one has $d_\mu \approx 0.3 \cdot 10^{-22} e \text{ cm}$. A d_μ in the $10^{-22} e \text{ cm}$ range is easily obtained and is compatible with bounds on $\tau \rightarrow \mu\gamma$ for reasonable values of $\epsilon_{23}, \epsilon_{32}, \tilde{\epsilon}_{23}, \tilde{\epsilon}_{32}$ (such that $a_\mu \sim 1$) and with bounds on $\mu \rightarrow e\gamma$ and d_e for an appropriate texture of λ_E with small 12 and 21 entries (such that $|a_e| \ll 1$). A d_μ in the $10^{-21} e \text{ cm}$ range requires in addition a moderately large $\tan \beta$ and accidental cancellations in $d_{\mu\tau}, d_{\tau\mu}$.

6 Conclusions

Motivated by the prospects of improving the experimental sensitivity to the muon EDM by several orders of magnitude, we have discussed the expectations for the muon and electron EDMs in supersymmetric scenarios. If the EDMs scale with masses, $d_\mu/d_e = m_\mu/m_e$, the present limit on d_e implies $d_\mu \lesssim 0.3 \cdot 10^{-24} e \text{ cm}$. We studied if a larger muon EDM, $d_\mu/m_\mu \gg d_e/m_e$, can be obtained. The generic answer is of course yes: the MSSM has ~ 100 free parameters and one just needs to assume that the appropriate ones (e.g. A_μ , the muon A -term) have a large complex phase. However this possibility is not appealing. It is well known that the SUSY-breaking soft terms must satisfy some highly constrained structure in order to give an acceptable phenomenology. When a scenario able to account for these constraints is considered, it is not obvious that large effects and deviation from naïve scaling can be obtained. For example, if the soft terms are universal one has $A_\mu \approx A_e$ and $m_{\tilde{e}} \approx m_{\tilde{\mu}}$ so that the relation $d_\mu/m_\mu \approx +d_e/m_e$ cannot be evaded. It is therefore important to understand if a detectable d_μ can be obtained in frameworks able to account for the tight constraints on the structure of supersymmetry breaking, like some appropriate flavor symmetry or a flavor blind mechanism of SUSY breaking.

The first case can be realized with an appropriate non-abelian flavor symmetry. If the soft terms are complex the typical prediction for the EDM ratio is $d_\mu/m_\mu \sim d_e/m_e$, so that d_μ/d_e could only be enhanced by accidental order one factors with respect to the naïve value m_μ/m_e . In particular, one gets $d_\mu/m_\mu = -d_e/m_e$ if the electron mass arises from the 12 or 21 elements of the lepton Yukawa matrix. If instead the lepton Yukawa matrix has negligible 12 or 21 entries, some contributions to d_e and d_μ would naturally give $|d_\mu/d_e| \gg m_\mu/m_e$, if they were dominant, as it happens in some cases (e.g. large $\tan \beta$ or $m_{\tilde{e},\tilde{\mu}} \gg m_{\tilde{\tau}}$). If the soft terms are real (or almost real, as suggested by bounds on EDMs) but have a non universal structure (like the one allowed by flavor symmetries), sizable EDMs are obtained from

the phases of the Yukawa matrices. The reason is that the complex flavor rotations that diagonalize the lepton masses do not diagonalize, at the same time, the slepton masses. In this case, a lepton Yukawa matrix with small 12 or 21 entries naturally enhances d_μ/d_e by suppressing d_e . Despite this suppression, d_e can saturate its experimental bound, and a d_μ as large as $10^{-22} e \text{ cm}$ can be obtained compatibly with all other bounds. Unacceptably large effects are avoided because the induced phases are typically small.

In the case of a flavor-blind mechanism of supersymmetry breaking, CP-violation can be imprinted in the initially universal soft terms by radiative corrections due to some higher energy physics. The top quark Yukawa coupling in SO(10) generates a sizable $\mu \rightarrow e\gamma$ decay rate and a sizable d_e : it also gives rise to $|d_\mu/d_e| \sim |V_{td}/V_{ts}|^2$ times unknown $\mathcal{O}(1/3 \div 3)$ Clebsch-Gordon factors. Only with favorable factors it gives a d_μ slightly above the planned sensitivity. If the lepton Yukawa matrix has 11, 13 and 31 texture zeros, these factors always combine to give $d_\mu/d_e = -m_\mu/m_e$. In the see-saw context, radiative effects are generated by the neutrino Yukawa couplings. Since they are uncertain, it is not impossible to obtain a sizable d_μ . However, the minimal see-saw structure suggested by neutrino data (with the additional assumption that one unknown Yukawa coupling is large enough) generates significant lepton-flavour violating effects in μ and τ , but without CP violation. Large CP-violating effects in μ and τ are instead naturally obtained in a minimal SU(5)-unified see-saw, where a d_μ up to about $10^{-23} e \text{ cm}$ is naturally obtained compatibly with all other bounds.

In both cases, the maximal value of d_μ is typically accompanied by a $\tau \rightarrow \mu\gamma$ rate close to its experimental bound. Finally, we also updated predictions for $\mu \rightarrow e\gamma$ rates and related processes in SU(5) and SO(10) models, and critically re-examined which ‘predictions’ are possible in the see-saw context.

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A EDMs of charged leptons at one loop

We are interested in EDMs generated by complex phases in fermion and sfermion mass matrices. We therefore assume that the gaugino masses, the μ and $B\mu$ terms are real. Only neutralino exchange contributes to EDMs of charged leptons at one loop, giving

$$d_\ell = \sum_{n,I} \frac{e\alpha_{\text{em}}}{4\pi \cos^2 \theta_W M_{N_n}} \text{Im} \left[\mathcal{U}_{\ell_L E_I} \mathcal{U}_{E_I \ell_R}^\dagger \right] H_{n\tilde{B}} (H_{n\tilde{B}} + \cot \theta_W H_{n\tilde{W}_3}) g \left(\frac{m_{E_I}^2}{M_{N_n}^2} \right), \quad (27)$$

where $g(r) \equiv [1 - r^2 + 2r \ln r]/2/(1 - r)^3$, $n = \{1, \dots, 4\}$, $H_{n\tilde{B}}$, $H_{n\tilde{W}_3}$ are elements of the 4×4 matrix H diagonalizing the neutralino mass matrix, $H^T M_N H = \text{diag}(M_{N_1} \dots M_{N_4})$, \mathcal{U} is the 6×6 unitary matrix diagonalizing the charged slepton mass matrix M_E^2 (written in the supersymmetric basis where the mass matrix of charged leptons is diagonal) as $\mathcal{U}^\dagger M_E^2 \mathcal{U} = \text{diag}(m_{E_1}^2 \dots m_{E_6}^2)$.

In order to obtain a more useful expression, we diagonalize the 6×6 slepton mass matrix by treating the A -terms and the lepton masses as perturbations. In the approximation in which the first two slepton families are degenerate, we then find for the dipoles d_ℓ , $\ell = e, \mu$,

$$d_\ell = - \frac{e\alpha_{\text{em}}}{4\pi \cos^2 \theta_W} \text{Im} \left[A_2^\ell G(\tilde{\tau}_R - \tilde{\ell}_R, \tilde{\tau}_L - \tilde{\ell}_L) + \right. \\ \left. + A_{1L}^\ell G(\tilde{\tau}_R - \tilde{\ell}_R, \tilde{\ell}_L) + A_{1R}^\ell G(\tilde{\ell}_R, \tilde{\tau}_L - \tilde{\ell}_L) + A_0^\ell G(\tilde{\ell}_R, \tilde{\ell}_L) \right], \quad (28)$$

where

$$G(a, b) \equiv \sum_{n=1}^4 \frac{H_{n\tilde{B}}}{M_{N_n}^3} (H_{n\tilde{B}} + \cot \theta_W H_{n\tilde{W}_3}) g\left(\frac{m_a^2}{M_{N_n}^2}, \frac{m_b^2}{M_{N_n}^2}\right), \quad g(a, b) \equiv \frac{g(a) - g(b)}{a - b}$$

$$G(a - a', b) \equiv G(a, b) - G(a', b), \quad G(a, b - b') \equiv G(a, b) - G(a, b'),$$

$$\begin{aligned} A_{1R}^\ell &= v \cos \beta \tilde{A}_{\ell_R \tilde{\tau}_L} V_{\tilde{\tau}_L \ell_L}^\dagger, & A_0^\ell &= v \cos \beta \tilde{A}_{\ell_R \ell_L}, \\ A_{1L}^\ell &= v \cos \beta V_{\ell_R \tilde{\tau}_R} \tilde{A}_{\tilde{\tau}_R \ell_L}, & A_2^\ell &= v V_{\ell_R \tilde{\tau}_R} (\cos \beta \tilde{A}_{\tilde{\tau}_R \tilde{\tau}_L} + \mu \sin \beta \lambda_{\tilde{\tau}_R \tilde{\tau}_L}) V_{\tilde{\tau}_L \ell_L}^\dagger. \end{aligned} \quad (29)$$

Here, λ is the charged lepton Yukawa matrix and \tilde{A} are the corresponding trilinear soft terms. The flavor basis in which they are written is identified by their indexes ($\ell_{L,R}$, $\tau_{L,R}$ denote the lepton mass eigenstates, whereas $\tilde{\ell}_{L,R}$, $\tilde{\tau}_{L,R}$ denote the slepton mass eigenstates). The matrices V measure the lepton-slepton mixing. In terms of the unitary matrices U_L, U_R, T_L, T_R defined in eq. (17), we have

$$\begin{aligned} \tilde{A}_{\ell_R \ell_L} &= (U_R \tilde{A}_E U_L^\dagger)_{\ell_R \ell_L}, & \tilde{A}_{\tilde{\tau}_R \ell_L} &= (T_R \tilde{A}_E U_L^\dagger)_{\tilde{\tau}_R \ell_L}, & \tilde{A}_{\ell_R \tilde{\tau}_L} &= (U_R \tilde{A}_E T_L^\dagger)_{\ell_R \tilde{\tau}_L} \\ \tilde{A}_{\tilde{\tau}_R \tilde{\tau}_L} &= (\tilde{A}_E)_{\tilde{\tau}_R \tilde{\tau}_L}, & V_{\ell_L \tilde{\tau}_L} &= (U_L T_L^\dagger)_{\ell_L \tilde{\tau}_L}, & V_{\ell_R \tilde{\tau}_R} &= (U_R T_R^\dagger)_{\ell_R \tilde{\tau}_R} \end{aligned} \quad (30)$$

If $\tan \beta$ is so large that $m_\tau \tan \beta \sim m_{\tilde{\tau}}$ one needs a more complicated expression, but there is no qualitative change.

B One loop effects in SO(10) and SU(5) unified models

In SO(10) models, the unified top quark Yukawa coupling induces lepton flavor violations in left-handed and right-handed slepton mass matrices. Eq. (28) can be further simplified taking into account the peculiar structure of the flavor-violating A -terms generated by SO(10) effects (summarized in eq. (28) of [10]). The relevant penguin dipoles $d_{\ell\ell'}$ (defined as in eq. (26)) are the ones that involve the first two generations, given by $d_{\ell\ell'} = V_{\ell_R \tilde{\tau}_R} V_{\ell'_L \tilde{\tau}_L} V_{\tau_R \tilde{\tau}_R}^* V_{\tau_L \tilde{\tau}_L}^* F$ where

$$\begin{aligned} F &= \frac{\alpha_{\text{em}} m_\tau}{4\pi \cos^2 \theta_W} \left\{ (A_e + \mu \tan \beta) G(\tilde{e}_L, \tilde{e}_R) + (A_\tau + \mu \tan \beta) G(\tilde{\tau}_L, \tilde{\tau}_R) + \right. \\ &\quad \left. - \left(\frac{A_e + A_\tau}{2} + \mu \tan \beta \right) [G(\tilde{\tau}_L, \tilde{e}_R) + G(\tilde{e}_L, \tilde{\tau}_R)] \right\}. \end{aligned} \quad (31)$$

In SU(5) models, the unified top quark Yukawa coupling induces lepton flavor violations in the right-handed slepton mass matrix. The transition dipoles $d_{\ell\ell'}$ can give rise to detectable $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ rates, but no sizable lepton EDMs are induced. The transition dipoles that can induce significant effects are $d_{e\mu} = V_{\mu_R \tilde{\tau}_R}^* V_{e_R \tilde{\tau}_R} F$ and $d_{\mu\tau} = V_{\tau_R \tilde{\tau}_R}^* V_{\mu_R \tilde{\tau}_R} F$ where now

$$\begin{aligned} F &= \frac{\alpha_{\text{em}} m_\mu}{4\pi \cos^2 \theta_W} \sum_{n=1}^4 \left\{ \frac{H_{n\tilde{B}}^2}{M_{N_n}^2} \left[f\left(\frac{m_{\tilde{\tau}_R}^2}{M_{N_n}^2}\right) - f\left(\frac{m_{\tilde{e}_R}^2}{M_{N_n}^2}\right) \right] + \frac{H_{n\tilde{B}}}{M_{N_n}^3} (H_{n\tilde{B}} + \cot \theta_W H_{n\tilde{W}_3}) \times \right. \\ &\quad \times \left[(A_e + \mu \tan \beta) g\left(\frac{m_{\tilde{e}_L}^2}{M_{N_n}^2}, \frac{m_{\tilde{e}_R}^2}{M_{N_n}^2}\right) - (A_\tau + \mu \tan \beta) g\left(\frac{m_{\tilde{e}_L}^2}{M_{N_n}^2}, \frac{m_{\tilde{\tau}_R}^2}{M_{N_n}^2}\right) \right] + \\ &\quad \left. - \frac{H_{n\tilde{B}} H_{n\tilde{d}}}{M_{N_n} M_Z \cos \beta \sin \theta_W} \left[g\left(\frac{m_{\tilde{\tau}_R}^2}{M_{N_n}^2}\right) - g\left(\frac{m_{\tilde{e}_R}^2}{M_{N_n}^2}\right) \right] \right\}. \end{aligned} \quad (32)$$

and $f(r) = -[2 + 3r - 6r^2 + r^3 + 6r \ln r]/[6(r - 1)^4]$.

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